$$E_V = \frac{1}{2} m_V e^2$$

is the energy stored for uniform uniaxial strain and

$$E = \frac{1}{2} m e^2$$

is the actual energy stored, the difference,

$$\delta E = \frac{1}{2} (m_V - m) e^2 ,$$

is the energy stored in random strain. For simplicity, an upper bound for this random strain energy can be obtained by replacing m with m_R. Call this upper bound $\overline{\delta E}$. An upper bound for the ratio of random strain energy to uniform uniaxial strain energy is

$$\frac{\overline{\delta E}}{E_V} = \frac{m_V - m_R}{m_V} .$$

From Equation (V.1) through Equation (V.6), this is approximately

$$\frac{\overline{\delta E}}{E_V} = \frac{1}{\left(1 + \frac{3}{4}\frac{K}{\mu}\right)} \frac{(S - 1)^2}{\left(1 + \frac{2}{3} S\right)\left(1 + \frac{3}{2} S\right)} \tag{V.8}$$

where

$$S = \frac{2c_{44}}{c_{11} - c_{12}}$$

is a measure of the isotropy of the material. (This is exact for K and μ replaced by K_V and μ_V but the difference is negligible.)

Since the energy is proportional to the square of the strain, an upper bound measure of the ratio of the random strain to the uniform strain is

$$\frac{\overline{e}_{(\text{random})}}{e_{(\text{uniform})}} = \left[\frac{1}{\left(1 + \frac{3}{4}\frac{K}{\mu}\right)} \frac{(S - 1)^2}{\left(1 + \frac{2}{3}S\right)\left(1 + \frac{3}{2}S\right)}\right]^{1/2} . \qquad (V.9)$$

For YIG, $\mu = 0.78 \times 10^{12}$ dynes/cm^2, $K = 1.62 \times 10^{12}$ dynes/cm^2, and $S = 0.95$. This gives

$$\frac{\overline{e}_{(\text{random})}}{e_{(\text{uniform})}} = 0.0015.$$

Copper is an example of a highly anisotropic cubic material. $\mu = 0.436 \times 10^2$ dynes/cm^2, $K = 1.33$ dynes/cm^2, and $S = 3.2$. For copper,

$$\frac{\overline{e}_{(\text{random})}}{e_{(\text{uniform})}} = 0.28.$$

It was stated that this was an upper bound. A better estimate can be made by using Equation (V.7) in the analysis. The result differs from Equation (V.9) by a factor of $1/\sqrt{2}$. This gives a strain ratio in copper of 0.20.

This calculation shows that the assumption of uniform strain in YIG is quite good. However, for highly anisotropic material the deviation from uniform strain can be quite appreciable.